Chapter 17

# *Synchronous integration*

## *17.1 'Boxcar' detection systems*

Phase sensitive detection systems are ideally suited to dealing with signals which have a steady, or relatively slowly varying, level. In many situations, however, we need to measure the details of a signal which varies quite swiftly in a complex manner. The signal may also not last very long. In order to measure brief, rapidly changing signals a different approach is required. *Synchronous Integration* is a technique which allows measurements to be made on complex signal patterns which have powers well below the general detector or amplifier noise level. The technique can be employed in various ways provided two basic requirements are obeyed. Firstly, the signal must be repeatable so we can produce a series of nominally identical pulses or *Signal Cycles*. Secondly, we must obtain an extra *Trigger* signal — similar to the phase reference signal required for a PSD — which can be used to tell the measurement system when each signal cycle begins. Although it's usually convenient to arrange for signal cycles to occur with a steady repetition rate, this isn't absolutely necessary provided we know when each cycle starts.

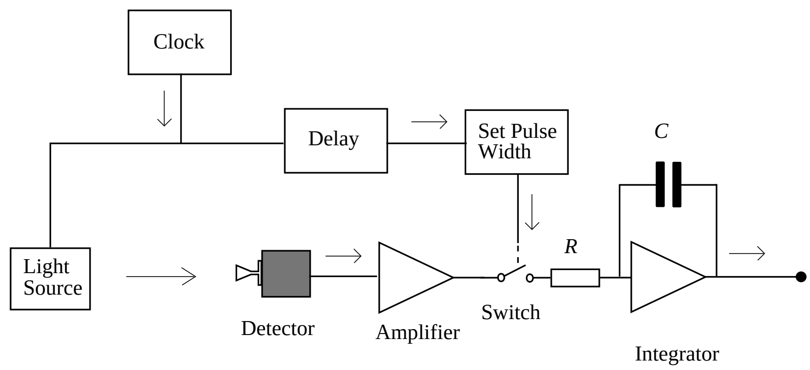


**Figure 17.1** Analog synchronous integration (boxcar) system.

These requirements are often satisfied by using some form of *clock* which regularly initiates the signal and provides the trigger information. Alternatively, the signal generating process may, in itself, provide some information telling us when each signal cycle begins. For the sake of illustration we can concentrate upon a situation where we wish to measure how the output light intensity of a pulsed laser varies with time during each output signal pulse. The techniques described in this chapter can, however, be applied to measure the shape of any repetitive signal pattern.

Some electrical gas discharge lasers can be arranged to produce a series of light pulses when connected, via a suitable circuit, to a steady power supply. Each burst of light output is accompanied by an abrupt drop in the voltage across the gas tube. Under these circumstances we could use the sudden fall in voltage to trigger the measurement process. More generally, however, we will have to provide some kind of clock signal to initiate light output. Figure 17.1 illustrates a typical system designed to measure how the output intensity of a pulsed laser varies with time. In this case we have arranged for the system to be controlled by a clock which both 'fires' the laser and triggers the measurements.

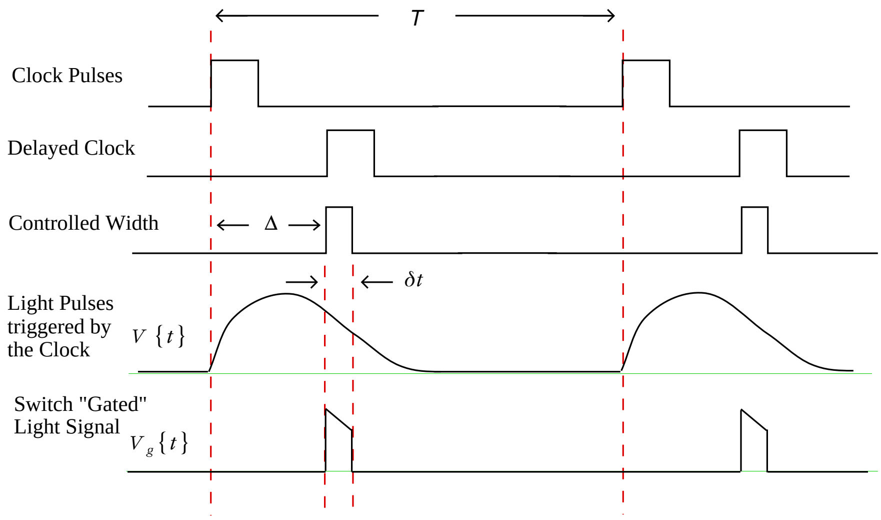


**Figure 17.2** Control and data waveforms in 'boxcar' integrator.

For the sake of simplicity we can assume that the clock which starts each cycle of light output has a period, *T*. This means that the resulting signal cycles will occur at the rate, $1/T$. Each clock pulse immediately starts a signal cycle. The clock also controls the operation of a switch which can connect the amplified signal to an analog integrator. The switch is only closed for a brief *Sampling Interval*, $\delta t$, which begins after a time delay, $\Delta$, following the appearance of each clock pulse.

Synchronous integration works on the basis that all the signal cycles are similar to one another. We can then define the shape of each individual pulse in terms of the same function, $v\{t\}$, where $t$ represents the time from the beginning of each signal cycle. Figure 17.2 illustrates a typical set of pulse and signal patterns we might see in a working system of this kind. The output voltage from the detector is amplified to produce a signal voltage, $V\{t\}$, which is presented to the switch. Since the switch is only connected for a brief period, $\delta t$, after a delay, $\Delta$, following the start of each clock pulse, the signal presented to the integrator looks like the waveform, $V_g\{t\}$, shown in figure 17.2. This can be defined as

$$V_g\{t\} \equiv V\{t\} \text{ when } \Delta \leqslant t \leqslant \Delta + \delta t$$

$$\text{otherwise } V_g\{t\} \equiv 0 \qquad \text{... (17.1)}$$

We can now start with the integrator (capacitor) voltage set to zero and allow the system to operate for $n$ signal cycles. In the absence of any noise this will produce an output voltage

$$V_o\{\Delta, \delta t\} = nK \int_0^T V_g\{t\}\, dt = nK \int_\Delta^{\Delta + \delta t} V\{t\}\, dt \qquad \text{... (17.2)}$$

where

$$K = \frac{-1}{RC} \qquad \text{... (17.3)}$$

and $R$ and $C$ are the values of the resistor and capacitor used in the analog integrator. The minus sign is present because an analog integrator normally reverses the sign of the signal (see Chapter 15). Provided $\delta t$ is sufficiently small, the signal level will not change a great deal between the times, $\Delta$ and $\Delta+\delta t$, and we can approximate the above integral to say that

$$V_o\{\Delta\} = nKV\{t\}\delta t \qquad \text{... (17.4)}$$

i.e., $V_o\{\Delta\}$, is proportional to the signal voltage, $V\{t\}$, which arises at a time, $t = \Delta$, following the start of each pulse. The output is also proportional to $nK\delta t$, hence we may increase the magnitude of $V_o\{t\}$ by operating the system for more clock cycles, increasing the value of $n$. In effect the system adds up the contributions from a series of pulses to magnify the output signal level.

In practice, the required signal will always be accompanied by some unwanted noise voltage, $e\{t\}$, which — being random — will differ from one pulse to another. This will contribute an unpredictable amount

$$E_o = K \sum_{i=1}^{n} \int_{\Delta}^{\Delta + \delta t} e\{iT + t\} \, dt \qquad \ldots (17.5)$$

to the integrated output voltage, where $e\{iT + t\}$ represents the noise voltage during the $i$ th pulse at a time, $t$, from its start.

Unlike the signal, these noise voltages which occur during each cycle are not all identical. As the noise is random in nature we can't say what value this error voltage will have when we make a particular measurement. As with all random quantities we can only predict the average, typical, or likely properties of the noise. Taking the simplest example of a ‘white’ noise input spectrum whose noise power spectral density is $S$. We can use the arguments presented in section 15.2 to say that the mean noise power added to a single integration will be $N_i = K^2 S \delta t / 2$. (This result comes from considering expression 15.9 and recognising that, in this case, the integration constant $K^2 \equiv 1 / \tau^2$.) This means that the voltage produced by each individual sample integration will typically differ from the next by a rms amount

$$\varepsilon_n = \sqrt{N_i} = K \sqrt{\frac{S \delta t}{2}} \qquad \ldots (17.6)$$

The noise power spectrum of a real white noise source can never extend over an infinite frequency range. (If it did, its total power would be infinite!) For a practical noise source we can therefore say that the input total noise power will be $N_{in} = SB_n$, where $B_n$ represents the *Noise Equivalent Bandwidth* of the input noise spectrum. Here we can assume that this means that the noise covers the frequency range from around d.c. (0 Hz) up to a maximum frequency equal to $B_n$. The input will therefore exhibit an input noise voltage level equivalent to an rms voltage of $e_n = \sqrt{SB_n}$.

Combining these expressions we can therefore say that the input and output rms noise voltage levels will be such that

$$\varepsilon_n = K e_n \sqrt{\frac{\delta t}{2 B_n}} \qquad \ldots (17.7)$$

This expression links the rms noise level, $\varepsilon_n$, at the integrator's output to the input level, $e_n$. We can now use this expression to determine the accuracy of a measurement using the synchronous integrator, although it is worth remembering that, in general, the precise relationship between $\varepsilon_n$ and $e_n$ depends upon the details of the input noise spectrum. A more detailed analysis would show that expression 17.7 is only strictly true for a noise spectrum which has a uniform noise power spectral density over a frequency range, $f_{min}$ to $f_{max}$ where $f_{min} \ll \frac{1}{2\delta t}$ and $f_{max} \gg \frac{1}{2\delta t}$.

As the actual noise level varies randomly from one measurement to another we can say that typical measured levels after $n$ signal cycles will be

$$V_o'\{\Delta\} = nKV\{\Delta\}\delta t \pm \varepsilon_n\sqrt{n} \qquad \text{... (17.8)}$$

The unpredictability of the noise means we can't predict a precise value for *V*. Instead, expression 17.8 indicates the most probably result, plus or minus the probable range of uncertainty. Here the prime indicates a typical measured value which may not exactly equal the result we might predict using expression 17.4. Combining expressions 17.4, 17.7 and 17.8 we can obtain

$$V_o'\{\Delta\} - V_o\{\Delta\} = \pm K e_n\sqrt{\frac{n\delta t}{2B_n}} \qquad \text{... (17.9)}$$

In effect this shows the probable difference between the values we would measure with and without random noise.

From expression 17.4 we could expect — in the absence of any random noise — to find the input signal voltage level, $V\{t\}$ at a time $t = \Delta$ from the expression

$$V\{t\} = \frac{V_o\{\Delta\}}{nK\delta t} \qquad \text{... (17.10)}$$

unfortunately, the inevitable presence of some noise means that a typical measurement leads to the actual result

$$V'\{t\} = \frac{V_o'\{\Delta\}}{nK\delta t} \qquad \text{... (17.11)}$$

Combining expressions 17.9–17.11 we can say that our measurement of the input voltage at any time will be

$$V'\{t\} = V\{t\} \pm e_n\sqrt{\frac{1}{2nB_n\,\delta t}} \qquad \text{... (17.12)}$$

From 17.12 we see that the accuracy of measurements of the input signal level will tend to improve as we increase the number of signal cycles we integrate over. Two points about this result are worth noting. Firstly, both the total input noise level <u>and</u> the frequency range it covers affect the accuracy of the measurement. This can be understood by imagining a situation where a given fixed total input noise power is 'stretched out' to cover a wider frequency range. The effect of such a change would be to move some of the noise power up to higher frequencies which find it more difficult to pass through an integrator. Hence the fraction of the noise which influences the output will fall if $B_n$ is increased while $e_n$ is kept constant. Secondly, the above result indicates the relative sizes of the measured signal and noise voltages. When considering the performance of a signal processing system in terms of S/N ratios we normally consider a power ratio. Since the voltage accuracy obtained above varies as $\sqrt{\delta t\, n}$ we can expect the output S/N (power) ratio provided by a synchronous integration system to improve with $\delta t\, n$ — i.e. in proportion with the number of signal cycles integrated.

In order to measure the overall shape of the signal waveform — and hence the way the laser intensity varies with time — we can now proceed as follows:

Firstly, set Δ to a particular value, zero the integrator voltage, and perform an integration over $n$ clock cycles. Note the integrator output level, increment Δ by an amount, $\delta t$, and rezero the integrator. Integrate again for $n$ cycles, and note the new output level. Repeat this process until a series of $V_o'\{\Delta\}$ values have been gathered which cover the whole of the signal cycle. Then use expression 17.11 for a set of times, $t = \Delta$, to determine the shape of the input signal with an accuracy which can be estimated using expression 17.12.

This form of measurement system is called a synchronous integrator because we perform integrations on samples which are *synchronised* with the signal cycles. Many of the earliest system employed an output time-constant instead of an integrator. The time delay, Δ, was then slowly swept continuously over the range 0 to $T$ and the smoothed output displayed on an oscilloscope or drawn on a plotter. These systems came to be called ‘boxcar’ integrators because the switch control pulse looked on an oscilloscope like an American railroad waggon running along a track.

Synchronous integration systems are very effective at recovering information about weak pulses when the noise level is quite high. As usual, however, there is a price to be paid for this improvement in the measured S/N ratio. The total measurement for any particular delay, Δ, takes a time $nT$ since we have to add up the effects of $n$ clock cycles. Hence when we improve the S/N ratio by increasing $n$, the measurement takes longer. A drawback of the method considered so far is that most of the time the output integrator is disconnected from the input! Only that fraction, $\delta t/T$, of the pulses which occur while the switch is closed contributes to the measurement result. As a consequence, to measure all the details of the pulse shape we have to repeat the measurement process up to $T/\delta t$ times for each $\Delta$ value. Hence the time required to measure the whole signal shape will be $nT^2/\delta t$. If $n$ is large and $\delta t$ small, this can turn out to be quite a long while!

To improve the S/N ratio without increasing the total measurement time we could chose to increase, $\delta t$, the duration of each sample. Unfortunately we can't expect to observe any signal fluctuations which take place in a time-scale less than $\delta t$ because they will be smoothed away by the integrator. When using a synchronous integrator we can only clearly observe details of the pulse shape which persist for a time $\geqslant \delta t$. We can therefore reduce the total measurement time by increasing $\delta t$, but this may mean that we can no longer see all of the fine details of the signal. Any real signal will only contain frequency components up to some finite maximum frequency, $f_{max}$. From the arguments outlined in chapter 2 (section 4) we can expect that we will only be able to see all the details of the signal when

$$\delta t \leqslant \frac{1}{2f_{max}} \qquad \text{... (17.13)}$$

In practice, therefore, $\frac{1}{2f_{max}}$ usually represents the optimum choice for $\delta t$. A smaller value increases the required measurement time, a larger value prevents us from observing all the details of the signal.

## 17.2 Multiplexed and digital systems

The system we have considered so far isn't a very efficient one since, in general, most of the signal power was ignored because it arrived when the switch was open. This problem can be dealt with by employing a *Multiplexed* arrangement.

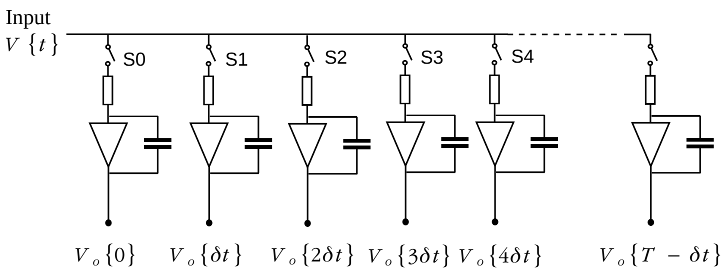


**Figure 17.3** Multiplexed array of synchronous integrators.

Figure 17.3 illustrates a multiplexed analog synchronous integration system. This works in a similar way to the one we have already considered, but it contains a 'bank' of similar switches and integrators. In this system the first switch, S0, is closed during the periods when $0 < t \leqslant \delta t$, S1 when $\delta t < t \leqslant 2\delta t$, S2 when $2\delta t < t \leqslant 3\delta t$, etc. By using an array of $M$ such switches and integrators, where $M = T / \delta t$, we can arrange that at <u>any</u> time during each pulse one or another of the switches will be closed and the signal is being integrated somewhere. At a time, $t$, during each pulse the $j$th switch will be closed, where $j$ can be defined as the integer value (i.e. the 'switch number') such that $j\delta t \leqslant t < (j+1)\delta t$. Each switch/integrator provides a separate sampling and integration *channel*.

The simple system we considered earlier had just one channel and could only look at a small part of the signal pulse at a time. The fully multiplexed version has $T / \delta t$ channels and covers the whole signal cycle. The system essentially produces a series of integrated output voltages, $V_o\{0\}$, $V_o\{\delta t\}$, etc, and gathers information about all the pulse features 'in parallel'. The advantage of this arrangement is that <u>all</u> of the information from each signal cycle is recorded by the bank of integrators. No signal information is wasted. As a result, the multiplexed system is much more efficient at collecting information than the single-channel version. Using this arrangement we don't have to keep repeating the integration process as $\Delta$ is varied.

Although multiplexing means that measurements can be made more quickly and efficiently, wholly analog systems of this type are now rarely used. This is partly because it can be difficult (and expensive) to arrange for a large number of nominally identical switches and analog integrators, but it is also because digital information processing techniques have advanced rapidly over the last few decades. Modern synchronous integration systems often use digital techniques to obtain, relatively cheaply, a level of usefulness it would be difficult to match using analog methods. As usual in information processing we can build various types of digital and analog systems to perform a given function. The system shown in figure 17.4 makes use of a circuit known as a *voltage to frequency convertor* (VFC) to implement a digital synchronous integration system. This is a device which produces an output square wave (or stream of pulses) whose frequency or 'pulse rate' is proportional to the input voltage. At any time, $t$, we can therefore expect the VFC to be producing pulses at a rate

$$f\{t\} = k_f V\{t\} \qquad \text{... (17.14)}$$

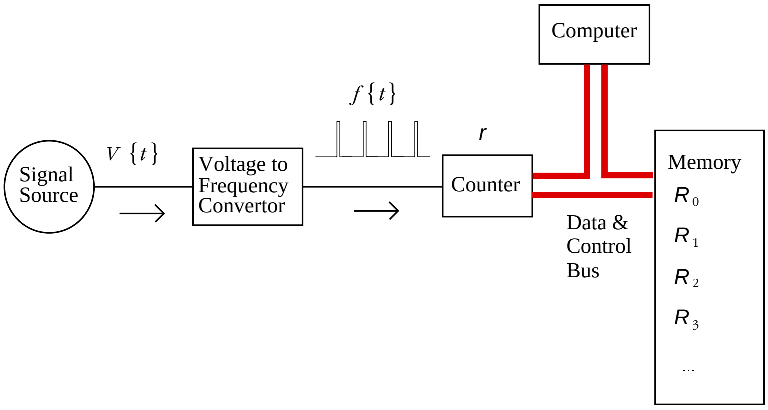


**Figure 17.4** Example of a digital system for performing multiplexed synchronous integration of a repetitive waveform.

where $k_f$ is a coefficient whose value depends upon the details of the VFC circuit being used. The operation of this system depends upon how we have programmed the computer. At the start of a measurement the computer should 'clear' (i.e. set to zero) the numbers stored in the parts of its memory which it will use for data collection. The computer then waits until it receives a trigger from the clock which is initiating the pulses to be measured (this can, if we wish, be the computer's own internal clock). The computer then proceeds as follows:

Firstly, the counter reading is zeroed. It is then allowed to count pulses coming from the VFC for a time, $\delta t$, and the resulting number, $r_0$, is added into a memory location at some *address*, $A_0$. The counter is then re-zeroed, allowed to count for another period, $\delta t$, and the new result, $r_1$, added into a memory location, $A_1$. This process is repeated over and over again until the whole signal cycle time, $T$, has elapsed. After one signal cycle the system will have stored a set of binary numbers, $r_0$, $r_1$, etc, in its memory. Each number will be approximately equal to

$$r_j = k_f \int_{j\delta t}^{(j+1)\delta t} V\{t\}\, dt \qquad \text{... (17.15)}$$

i.e. each number is proportional to the input voltage integrated over a short period of time. We can now repeat this process $n$ times to obtain a stored set of numbers, $R_0$, $R_1$, ... , which, in the absence of any noise, will approximate to

$$R_j = N r_j = n k_f \int_{j\delta t}^{(j+1)\delta t} V\{t\}\, dt \qquad \ldots (17.16)$$

In effect, these stored numbers are proportional to the integrated signal voltages at various times from the start of each signal cycle. They contain the same information about the signal pattern as we could have collected with an analog synchronous integration system. As with the analog system, if we arrange for $\delta t$ to be small enough we can approximate the above integral to

$$R_j = n k_f \delta t\, V\{t_j\} \qquad \ldots (17.17)$$

where $t_j = j\,\delta t$. We can therefore use the collected $R_j$ values to determine the signal voltage at various times during each signal cycle.

The counted values are a digital equivalent of the voltages collected at the output of a bank of analog integrators. Equation 17.17 is the 'digital equivalent' of expression 17.4 for an analog system. Each count is proportional to the input at the appropriate moment, $V\{t_j\}$.

This digital approach has a number of advantages over the analog technique. One particular advantage of the digital approach is that it is relatively easy to buy and use a large amount of computer memory. For example, we can imagine buying and using a single digital memory chip capable of holding 128 *kilobytes* of information. If we allocate 16 bits (i.e. two 8-bit bytes) to hold each $R_j$ we can store a set of values which represent integrated level measurements of the input signal shape at 64 × 1024 = 65,536 moments during each pulse. As a result, one cheap digital memory chip can replace over 65 thousand separate analog integrators!

# Summary

You should now understand how *Synchronous Integration* allows us to recover the details of a weak, transient phenomenon by adding together the information from a synchronised sequence of similar transient events. That a *Multiplexed* system allows us to avoid the signal information losses we get with a 'single integrator' system which tends to ignore most of the signal most of the time. That we can build either analog or digital systems to perform synchronous integration. You should now also see that the combination of a *Voltage to Frequency Convertor* and a *Counter* act as a form of integrator.