Using Phase-Modulated Probe Signals to Recover Delays from Higher-Order Non-linear Systems

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Abstract

A popular method of identifying parameters of a non-linear system has been through the use of binary pseudo-noise sequences as probe signals.

This paper describes a phase-modulation technique that allows complex sequences to be embedded into probe signals and their resultant distortion after passing through a system, analysed. The advantage of this approach is that individual orders of system response can be examined. For some circumstances, a square wave carrier allows the probe signal to be implemented as a binary sequence.

We determine a signal delay as it passes through a non-linear system with polynomial response. For non-linear systems, the usual approach is to approximate this response as minor perturbations of a linear model (Volterra), to use stochastic inputs and model the system statistically (Wiener), or to use blind system identification techniques.

The use of pseudo-noise sequences significantly improves the signal to noise ratio and, through deliberate selection of probe sequences and their parameters, particular aspects of system response can be further analysed.

1. Introduction

In this paper, pseudo-noise sequences are used to probe non-linear systems by analysing the distortion of the measured probe signal from the system.

Beginning with an overview of the Volterra/Wiener kernel estimation methods in section 2, three sequences are described and compared in section 3 - each with near-ideal correlation properties - particularly as they are affected by higher order system responses.

The manner in which higher order interactions result in symbol permutations is shown in Section 4, and this is followed by Section 5, which shows one way in which sequences can be embedded into a probe signal. The carrier may be a typical signal received by the system. This way a typical system response can be ascertained. Section 6 ends with some simulation results, followed by the conclusion.

2. The Kernel Approach

A non-linear system can be analysed by separating its response *v(t)*, to probe signals *s(t)*, into *nth* order kernels (Volterra [15]). In a sampled system, the order of a system or kernel refers to the number of time lags that are multiplied together by a system element to form the response. For example, this response might be described using Volterra kernels up to second order as:

$$v(t) = L_0 + \sum_{k=1}^{T} L_1(k)s(t-k) + \sum_{k_1=1}^{T}\sum_{k_2=1}^{T} L_2(k_1,k_2)s(t-k_1)s(t-k_2) + ... \quad (1)$$

where L_0 is the zeroth–order kernel (a constant value), L_1 and L_2 are discrete first- and second-order kernels, and T is the system's memory in discrete time units.

A widely used alternative is the Wiener [16] kernel functional expansion, which is based on the system's response to white noise of a certain power.

Conceptually similar to the Volterra kernel, the system response becomes

$$w_{(n)}(t) = \sum_{k=1}^{n} v_k(t) \quad (2)$$

so that the n^{th} order Wiener kernel $w_{(n)}(t)$ is composed of Volterra kernels from 0 to n which represent the best fit in a mean squared error (MSE) sense from a library of Volterra kernels containing terms up to n^{th} order. Thus $w_{(n)}(t) = w_{(n-1)}(t)$ plus a correction term involving all orders up to n.

Instead of white noise, a system's response to pseudo-noise can be analytically determined for a given Wiener kernel. This allows us to propose several different models, analyse their response to pseudo-noise stimulation, and compare this to an experimental response to the same pseudo-noise input.

Judicious use of different types of pseudo-noise signals allows different elements of the system under study to be isolated.

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3. PN Sequences as Digital Signatures

The use of pseudo-noise (PN) sequences for system identification has been well established since the work of Briggs et al [3], and Barker et al [1].

System probing can be improved by embedding a 'signature' onto a typical system input signal, and to observe the distortions to the signature in the system response.

There are a number of ways to extract a digital signature from a system output. The most common of these is to cross-correlate the output signal against the phase-aligned input signature.

In our method, the most desirable feature of the signature is that its auto-correlation is two-valued, returning one value for in-phase and another for out-of-phase correlations. Another desirable feature is that cross-correlation with different members of the same 'family' of sequences is one- or two-valued.

We will describe three different sequences for system probing: the familiar M-sequence [10,11], the Legendre sequence [9,12], and a chirp-like Distinct Sums sequence [7].

3.1 The Generalised M-sequence

Zierler [17] introduced the concept of M-sequences and showed their advantages as a binary sequence. They are used in many areas for signal phase recovery in noisy environments like communication [10,6 - which describes these in detail], concert hall acoustics [12], digital image signature recovery [14] and as probe signals used in medicine [2,4].

M-sequences with alphabet α come in lengths of α^n-1, where α is prime and n is an integer, and can be generated by using a shift register capable of storing values from $0\ldots\alpha-1$, with feedback multiplication and summation modulo α. The polynomial coefficients are determined by the primitive polynomial for the field of length α^n. The sequence values $a_n(t) \in \{0,1,\ldots,\alpha^n-1\}$ are then spaced uniformly around the complex unit circle, $b_n(t) = \exp(2\pi i\, a_n(t) / \alpha)$.

Unfortunately, the cross-correlation of different M-sequences of the same length is poor (it giving multiple spurious peaks [13]).

3.2 The Generalised Legendre-sequence

Binary Legendre sequences have been used for over 30 years, appearing under various names as Quadratic Residue sequences [18], Uniformly Redundant Arrays (URA's) which are coded apertures and can be constructed from two-dimensional M-sequence or 'quadratic-residue arrays' [see for example, 5 and 8].

Generalised non-binary Legendre sequences were introduced by Schroeder [12]. A sequence of prime length p is defined as:

$$a_0 = 0, \quad a_n = \exp\left(\frac{2\pi i\; r\; ind_g t}{p-1}\right), \tag{3}$$

where $n = 1\ldots p-1$, $ind_g t$ is the number theoretic logarithm function (see [12] for details), and r is an integer $0\ldots p-2$ which distinguishes different sequences from the same family. The sequence alphabet is fixed for a length p to be $\alpha = gcd(p-1,r)$. Again, the sequence values are spaced uniformly around the complex roots of unity.

Normally $a_0 = 0$, but a special case exists for binary Legendre sequences of length $p = 4k-1$ with k an integer. In that case with $a_0 = -1$ and $r = (p-1)/2$, a_n will be a binary sequence.

A disadvantage of generalised Legendre sequences is that $ind_g\, t$ is inherently difficult to evaluate for long sequences [12], so long sequence generation is costly.

Although possessing even or odd symmetry (depending on $p = 4k + 1$ or $4k + 3$), each half of a sequence can be considered pseudo-noise [12].

3.3 The Distinct Sums Sequence

Chirp-like Distinct Sum (DS) sequences are a particular instance of a more general array construction method [7,14] which allows a 1D sequence with good correlation properties to be extended to higher dimensions while retaining these properties.

DS sequences are created by first building an ascending sequence of p^{th} roots of unity values:

$$a(n) = \exp(2\pi i\, n / p), \qquad n = 0 \ldots p-1 \tag{4}$$

and then shuffling the sequence elements to form the DS sequence, $b(k)$:

$$b(k) = a(n_k), \quad n_k = (n_{k-1} + rk) \bmod p, \quad n_0 = 0, k = 1 \ldots p-1 \tag{5}$$

where each r specifies a unique sequence. There are $r = 0\ldots p-1$ different sequences possible for a given length p.

Like M-sequences, long DS sequences are simple to construct in hardware, however unlike the other two sequences, DS sequences are not pseudo-noise.

4. Effects of Higher-order Interaction

While binary M-sequences and Legendre sequences appear to behave very similarly [4], this is not the case for the higher alphabet versions of each.

Consider a ternary sequence: with alphabet 1, α, α^2, all being roots of unity. We may express these values as angles on the complex unit circle

$$s_t = \exp(2\pi i t + \psi(t)) \tag{6}$$

where $\psi(t)$ is a time-varying phase offset. For a ternary sequence, it will take the values of $2n\pi/3$, *for* $n = 0, 1, 2$ corresponding to the $\{1, \alpha$ and $\alpha^2\}$ terms.

If the system has a 2nd order effect, there will be a component:

$$s_t^2 = \exp(4\pi i t + \psi'(t)) \quad (7)$$

in the response where $\psi'(t) = \psi(t)$ with all its symbols (angles) doubled yielding an alphabet $\{1, \alpha^2, \alpha^4\}$. For ternary signal, $\alpha^4 \ (mod\ 3) = \alpha$, there is a permutation of part of this sequence alphabet to $\{1, \alpha^2, \alpha\}$.

Similarly one might imagine a 3rd order effect for this signal producing the alphabet 1, α^3, α^6. However these all map onto 1. Turning this to advantage, it is possible to assign a field such that individual order effects (in this case, 3rd order) can be removed.

To maintain a one-to-one mapping of symbols, over an nth order transformation, it is necessary to use an alphabet greater than n and containing the least common multiple between all values of 1 to n. To study 4th order effects, for example, this means an alphabet of $lcm(2,3,4) = 12$ is needed.

The symbol permutation has different effects on the different sequences. Everett [18] shows that an M-sequence of length p^n is actually composed of $p-1$ blocks, each of length $(p^n-1)/(p-1)$ and each block is a constant angular multiple of the previous block. Thus, in a cross-correlation, if the response contains more than one order, each of these subsequences will partially correlate with each other, resulting in multiple peaks.

Raising the generalised Legendre or DSP sequence to a higher power is equivalent to changing r. Each higher order thus interferes minimally with other orders.

5. Phase Modulated Input Signals

Consider a non-linear polynomial-response system whose usual inputs are (for simplicity) sinusoidal. If we phase-modulate this signal with a PN sequence as described above, the higher-order effects will manifest themselves on the signal and the signature.

As shown in a possible model in figure 1, we can use the symbol permutation to advantage. The scheme above will only correlate against the s component of the response. The higher order S^n, $n > 1$, have each suffered a symbol permutation, so the cross-correlation $R(s^n,s)$ will depend on the sequence used.

For M-sequences, the subsequence effect described before will result in $R(s^n,s)$ yielding multiple peaks, however for the Legendre sequence, $|R(s^n,s)|$, $n \neq 1$ is two valued (0 for in-phase, otherwise $\sqrt{(p-1)}$). For DS sequences $|R(s^n,s)|$ is one valued ($\sqrt{p}$).

To isolate a particular order of system response, the correlation template is distorted for that order (by multiplying all angles and thus permuting symbols).

It is possible to use non-sinusoidal probe carriers, provided each frequency component of the carrier is separated sufficiently that there is no overlap in the frequency bands created by the phase modulation. If selectivity of order of response is not desired, the reference used to cross-correlate with the response could be a sine wave with the same fundamental frequency as the probe carrier.

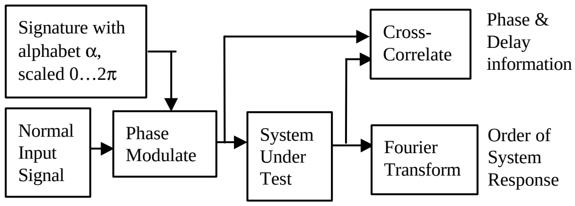

Figure 1. A block diagram of the proposed system measurement

6. Results

Figures 2 and 3 show some screen shots of the simulation system. The top window in Figure 2 shows the angles, scaled 0…1, of a 7-element Legendre sequence. The next window shows the carrier signal phase-encoded by the angles of the Legendre sequence. The window marked 'Poly Mod Carrier' shows the output of a polynomial-response system modelled as $y(t) = x(t-46)$. The a_0 term of the Legendre sequence is implemented as a 'drop' in the carrier for one 'chip' time.

The 'FFT' window demonstrates the bandwidth expansion caused by the phase modulation. Finally, the correlation window at the bottom shows the periodic cross-correlation between the signals shown as 'Mod Carrier' and 'Poly Mod Carrier' revealing a delay of 46 samples. A Hilbert envelope has been applied to precisely locate the delay.

Figure 3 shows the output for a 127-element Legendre sequence into a system with model $y(t) = x(t-123)^2 + x(t-46)$, plus a uniform noise level equal in amplitude to the signal. The system is set to show only the second-order delay.

7. Conclusion

A novel approach has been demonstrated to determine the time delay in a system with a non-linear response characterised by a polynomial of unknown order. The input signal is encoded with a suitable sequence in phase domain. This enables the separation of different orders in the non-linear response and thus independent delay measurement for each order of response.

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9. References

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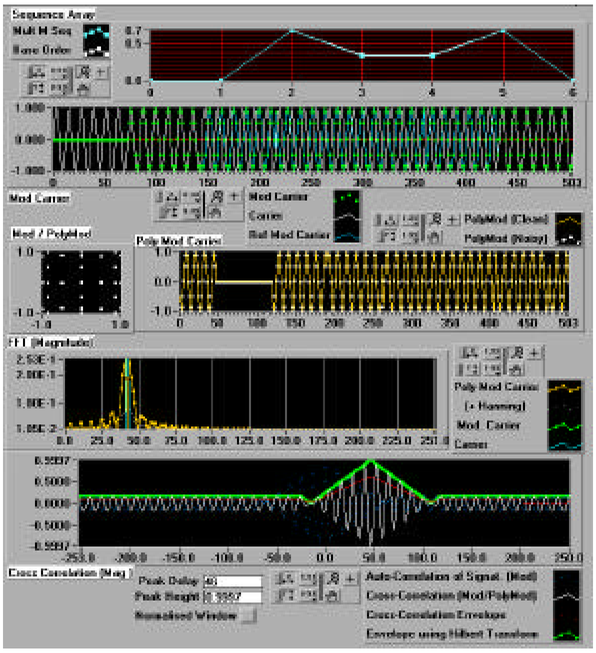

Figure 2. Screen shot of the system, showing the input sequence in the top window (an M-sequence of length 7), and the resultant correlation in the bottom window. A linear system y(t) = x(t-46) (i.e. a first order delay of 46 units) is shown. Note the Fourier transform of the system output shows the size of the frequency band.

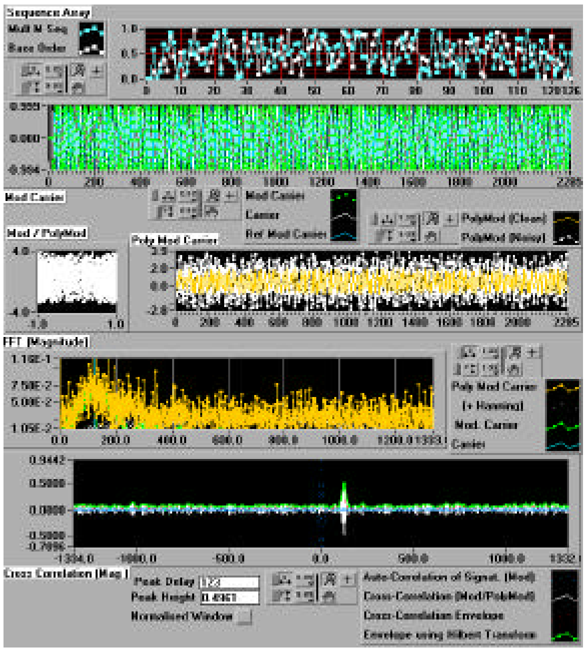

Figure 3. A larger sequence was used (resulting in a smaller correlation width) on a system $y(t) = x(t-123)^2 + x(t-46) + n(t)$, where n(t) is random noise equal in amplitude. A 127 element Legendre sequence with an alphabet of 126 was used. The lowest window shows the delay of 123 for the second order component without the subsequence peaks that an M-sequence would show.

In practise, much longer sequences would be used to handle the noise more effectively.