Application of Kalman Filter to Noise Reduction in Multichannel Data

ANDRZEJ LEŚNIAK[1], TOMASZ DANEK[1], MAREK WOJDYŁA[2]

[1]Department of Geoinformatics and Applied Computer Science
AGH University of Science and Technology, Mickiewicza 30, 30-059 Kraków, Poland
[2]PBG Geophysical Exploration Company Ltd., Jagiellońska 76,
03-301 Warszawa, Poland
e-mail: *lesniak@agh.edu.pl*

Abstract. The results of application of multichannel Kalman filtering to reduction of uncorrelated noise in magnetotelluric recordings are discussed in this article. Magnetotelluric method of Earth structure recognition is shortly presented together with the its most popular measurement method called the remote reference method. The basic theory of nonstationar, discrete Kalman filter and its implementation to multichannel magnetotelluric data recorded in multi-site experiment are also discussed with details. The practical examples of Kalman filter application to the real 2D and 3D data illustrate the merits of presented technique.

Keywords: Kalman filter, noise rejection, magnetotellurics.

1. Introduction

Application of multichannel Kalman filtering to reduction of uncorrelated noise in magnetotelluric data is discussed in the article.

Magnetotelluric recordings provide important information about structure of the Earth. They were successfully used to build or verify models that describe inner structure of the Earth. After specialized processing we can find so called transfer function that describes the conductivity distribution below the Earth surface. Usually the useful signal is buried in various kinds noises therefore the first and most important step in data processing is noise canceling.

Numerous methods of noise reduction in magnetotelluric data exist. They varies from simple deterministic band pass filtering to sophisticated methods that incorporate artificial intelligence methods. The first can be use to narrow frequency noise [2] while the other have larger scope of application [1].

The methods used so far to noise reduction of such data usually do not take advantage of the fact that many of magnetotelluric data (especially recorded in EM AP technique or multi-site survey) are multichannel. For the reason of that and because of the alternating spectrum and irregular appearance of the noise in magnetotelluric data we have to use adaptive filtering to solve the problem. We propose to use the Kalman filtration of reference recordings to effective noise reduction or even rejection.

2. Theory

The Kalman filter is a nonstationary, recursive filter that allows estimation of the useful signal in noisy time series in each moment of time [4]. In steady state Kalman approach to digital filtering signal $\{x_k\}$ is described by two linear difference stochastic equations:

$$x_k = F x_{k-1} + G w_{k-1},$$
$$y_k = H x_k + v_k. \tag{1}$$

The first equation describes process of signal generation while the second process of signal measurement. The random variables $\{w_k\}$ and $\{v_k\}$ represent the process and measurement noises (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions. The covariance matrix of process noise

$$E[w_k w_l] = Q_{kl} \qquad p(w_k) \sim N(0,Q) \tag{2}$$

and covariance matrix of measurement noise

$$E[v_k v_l] = R_{kl} \qquad p(v_k) \sim N(0,R) \tag{3}$$

might change with each time step of time series, however here we assume they are constant.

The matrix **F** in the first difference equation relates the state at the time step 'k–1' to the state at the step 'k', in the absence of either a driving function or process noise. The matrix **F** might change with each time step, but we assume it is constant. The matrix **H** in the measurement equation relates the state x_k to the measurement y_k. We also assume here that it is constant. The Kalman filter estimates the process state at step 'k' and then obtains feedback in the form of (noisy) measurements. Therefore, the equations for the Kalman filter fall into two groups. The first is time update equations:

$$\hat{x}_{k|k-1} = F\hat{x}_{k-1|k-1},$$
$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q \tag{4}$$

and the second measurement update equations:

$$K_k = P_{k|k-1}H_k^T\left[H_k P_{k|k-1}H_k^T + R\right]^{-1},$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(y_k - H_k\hat{x}_{k|k-1}\right), \tag{5}$$
$$P_{k|k} = \left[I - K_k H_k\right]P_{k|k-1}.$$

The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. Matrix $P_{k|k-1}$ is *a priori* estimate of error covariance:

$$P_{k|k-1} = E\left[\left(x_k - \hat{x}_{k|k-1}\right)\left(x_k - \hat{x}_{k|k-1}\right)^T\right], \tag{6}$$

while matrix $P_{k|k}$ is *a posteriori* estimate of error covariance

$$P_{k|k} = E\left[\left(x_k - \hat{x}_{k|k}\right)\left(x_k - \hat{x}_{k|k}\right)^T\right]. \tag{7}$$

As we can see from the second filter measurement update equation *a posteriori* state estimate $\hat{x}_{k|k}$ is a linear combination of an *a priori* estimate $\hat{x}_{k|k-1}$ and a weighted difference between an actual measurement y_k and a measurement prediction

$$\hat{y}_{k|k-1} = H_k\hat{x}_{k|k-1}. \tag{8}$$

Starting from the initial estimates $\hat{x}_{0|0}$ and $P_{0|0}$ one can obtain estimates of vector $\hat{x}_{k|k}$ for any time 'k'.

3. Magnetotelluric observations and data

The advantages of multichannel Kalman filters can be shown in noise rejection in data recorded during measurements of natural electromagnetic field induced in the Earth.

Magnetotelluric measurements are passive measurements of fluctuations of natural electric and magnetic fields at the Earth's surface [6]. Such fluctuations of electromagnetic field generate so called magnetotelluric waves. Periods of magnetotelluric waves usually ranges from 10^{-3}s to 10^{5}s what depends on the wave source. The shortest waves are generated by lightning strikes while longer one by ionosphere resonance and solar wind variations. Penetration of magnetotelluric waves ranges from ~150 m to more than 500 km.

The essential aspect of interpretation procedure of magnetotelluric data is estimation of transfer function. A noise reduction is a critical moment of that procedure. An awkward noise in magnetotelluric time series can be generated by measurement equipment (especially sensors), meteorological phenomena and anthropogenic factors (industry installations and devices, railways). One of the fundamental techniques that allows the noise elimination is a remote reference method [3].

In this method components of the magnetotelluric field are synchronically registered in measurement and reference points. Application of appropriate processing procedures to this data allows uncorrelated noise to be reduced. The magnetic remote reference point has to be localized in a place free of disturbances. In practice only uncorrelated noise exists in data recorded in places located in distance larger than several dozens of kilometers. Such reference is called the distance remote reference. If multi-site measurement instrumentation is used only in relatively small area it is possible to use mutual electrical components of magnetotelluric field as a reference components. They are called local remote reference.

In practice the simplest method of the impulse-type disturbances reduction is applied. These spikes are simply rejected and the voids appears in recordings. It makes evaluation of the impedance function for low frequencies difficult and erroneous. The more appropriate method of noise reduction is a proposed Kalman filtering.

In contrary to frequency filtering the Kalman filter can effectively reject uncorrelated disturbances because it preserve the shape of noise-free registrations. Therefore the errors of impedance function estimation are significantly reduced. The filter enables also reduction of the noise uncorrelated between particular channels. As an output of the filter we obtain corrected electric or magnetic recordings consisted only from components

which exist in reference point. Consequently, the filter can reduce the atmospheric and industrial noises if their sources ware located close to measurement points.

The data used in our study was recorded in area where noise was relatively large comparing to other measurements in that region. Analyzed measurement point was localized in area where noise intensity was particularly high. The disturbances were caused by electricity power lines and transformer stations, railways, traffic and buildings. High-voltage lines were 350 meters away, electric railway lines were 1250 meters away and some inhabited buildings were in close vicinity of the point. Furthermore, some of the disturbances were qualified as a meteorological ones, generated by wind or lightings. The data gathered in measurement point consists of electric field recorded by three dipoles aligned in north-south direction (Ex1, Ex2, Ex3 components), three dipoles aligned in west-east direction (Ey1, Ey2, Ey3 components) and magnetic field recorded by two magnetometers aligned in north-south (Hx component) and east-west direction (Hy component). Magnetic field was also recorded in reference point by two magnetometers aligned in north-south (HxR component) and east-west direction (HyR component).

Recorded time series are very long. They typically consist of several millions of data points, because measurements in one point can take up to one week of continuous recordings. We present the results only from short window of 800 s length. The corresponding time series are shown in Fig. 1.
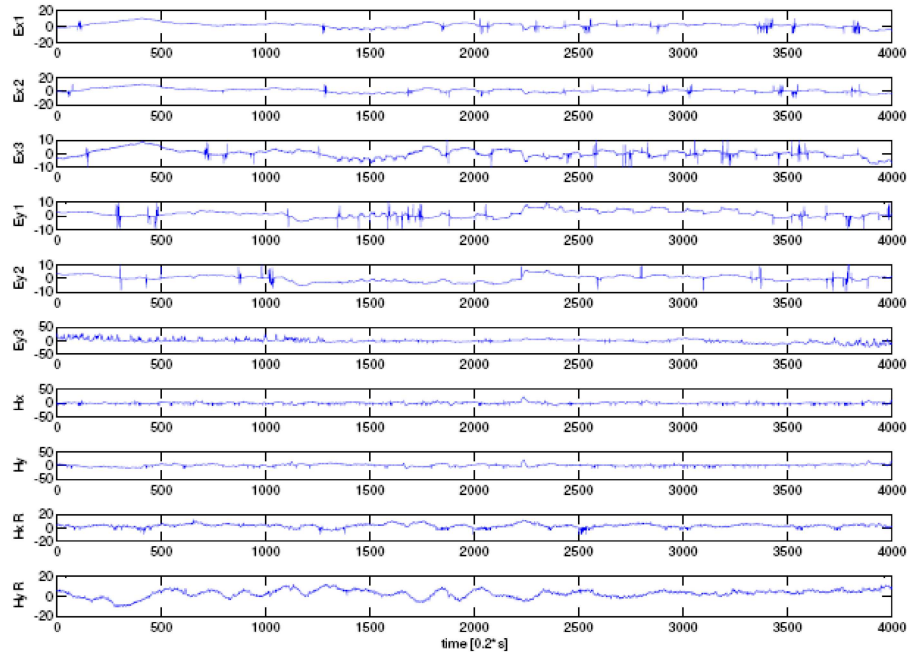
Fig. 1. Windowed magnetotelluric data of 800s length

4. Kalman filtering of multichannel magnetotelluric data

In remote reference method components of the magnetotelluric field are synchronically registered in measurement and reference points. We compare two or three components of the same type (electric or magnetic). We assume that each component consists of useful signal and noise, appropriate for each component, as shown below:

$$y_k^{(1)} = x_k + v_k^{(1)},$$
$$y_k^{(2)} = x_k + v_k^{(2)},$$
$$y_k^{(3)} = x_k + v_k^{(3)},$$

(9)

where $k = 0, 1, \dots$ – number of sample.

We built the Kalman filter in the following way [5]. As a reference signal for a first channel we used a signal recorded in second channel. In case of three component interpretation the useful signal for the first channel is an

average of the signals from second and third channel. The Kalman filters for two component filter is given as:

$$\begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{k-1}^{(1)} \\ x_{k-1}^{(2)} \end{bmatrix} + \begin{bmatrix} w_k^{(1)} \\ w_k^{(2)} \end{bmatrix}$$
$$\begin{bmatrix} y_k^{(1)} \\ y_k^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \end{bmatrix} + \begin{bmatrix} v_k^{(1)} \\ v_k^{(2)} \end{bmatrix} \qquad k = 0,1,\dots \qquad (10)$$

and for three component case as:

$$\begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} x_{k-1}^{(1)} \\ x_{k-1}^{(2)} \\ x_{k-1}^{(3)} \end{bmatrix} + \begin{bmatrix} w_{k-1}^{(1)} \\ w_{k-1}^{(2)} \\ w_{k-1}^{(3)} \end{bmatrix}$$
$$\begin{bmatrix} y_k^{(1)} \\ y_k^{(2)} \\ y_k^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k^{(1)} \\ x_k^{(2)} \\ x_k^{(3)} \end{bmatrix} + \begin{bmatrix} v_k^{(1)} \\ v_k^{(2)} \\ v_k^{(3)} \end{bmatrix} \qquad k = 0,1,\dots \qquad (11)$$

In this approach useful signal is estimated from measurement signal if noise in reference channels is uncorrelated. The starting covariance matrix is equal $P_{0p} = 0.1 * I$, where I is unit matrix. Different values for process noise covariance and measurement noise covariance were applied to the calculation. Their values were estimated in five point moving window centered in the point of evaluation "k".

Below we present the results of two components filtering for magnetic recordings (in measurement and reference points) evaluated in Matlab environment. For electric field recordings we used three component filter.

In Fig. 2A. the time series presents local and remote components of magnetic field for time window of length 400s. The signals were recorded in two distant points and differ in shape. Remote signal recorded in noise free environment (green curve) does not contain spikes which are frequent in signal recorded in local point (blue curve). After filtering local and remote signals are very similar (see Fig. 2B). The small differences are proportional to measurement noise $\{v_k\}$. In Fig. 3A. the measured and filtered signals in local point are presented (blue and green curves, respectively). Filtered signal is much smoother than the recorded one. As spectral analysis shows Kalman filter reduced the high frequency components of the recorded signal. Of course the frequency characteristics of the filter vary in time, because it depends on the reference signals. For the analysis window of length 2048 samples the difference between recorded and filtered signals shown in Fig. 3B is zero

mean, white noise, uncorrelated signal. For shorter analysis window these observations are not always true.
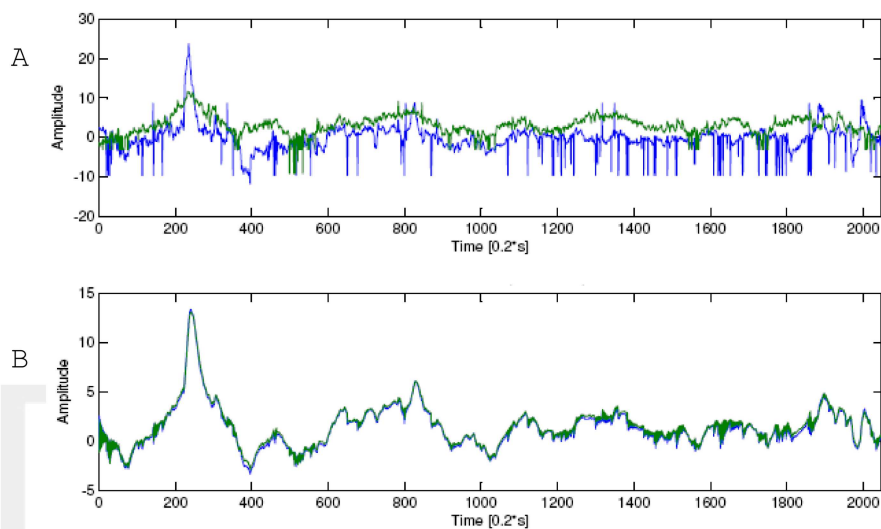
Fig. 2. Results of proposed Kalman filtration in case of local and remote magnetic components. Raw (A) and filtered (B) time series for channels Hx (blue), HxR (green)
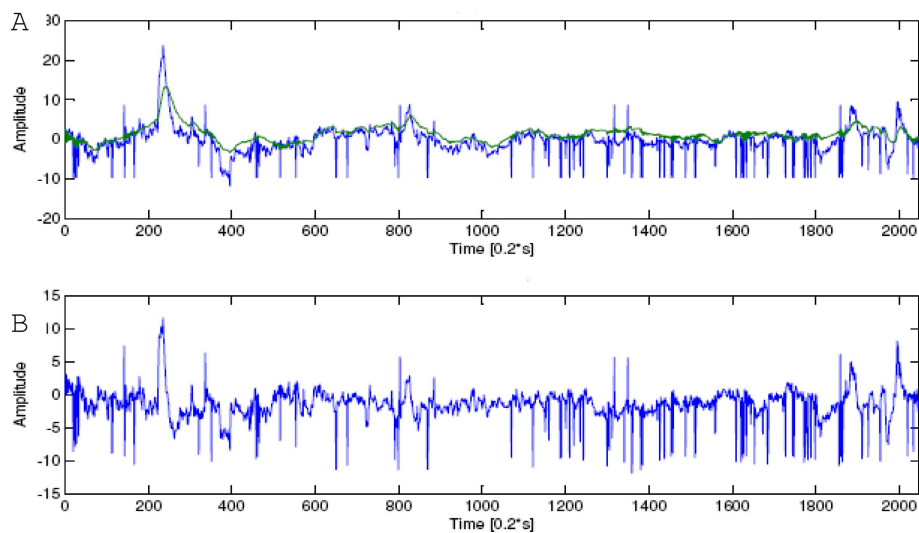
Fig. 3. A) Comparison between original (blue) and filtered (green) Hx time series in case of two component Kalman filter. B) Difference between raw and filtered time series

Electric components of the electromagnetic field are very similar because they are recorded close to each other (maximum distance is usually less than few hundreds of meters). As can be seen in Fig. 4A the time series differ in narrow spikes and some bias (especially seen in the first half of the series). Time series are mutually uncorrelated if we compare signals in different channels. The Kalman filtering of the electric components gives as a result very similar curves (see Fig. 4B). The filter removes almost perfectly uncorrelated noise (small field variations and narrow spikes). It also reduces the bias components.

If we compare measured and filtered signals we can see that coherent noise is not removed from the signal. For example artificial noise in time window from 1350 to 1700 samples in Fig. 4 (probably generated by passing train) is recorded in the similar way in each component in X direction. After filtering the amplitude of that noise is reduced but still measured and filtered signals are correlated (what is easily seen in Fig. 5A). Only the bias and narrow spikes that appear in different times in different channels are removed perfectly.

The difference between measured and filtered signals in case of electric components is not a white noise in narrow window but is still a zero mean process (Fig. 5B).

5. Discussion and conclusions

The proposed method of noise removal does not introduce any quantitative information about the noise and useful signal. It also do not use any *a priori* parameters (such as filter type ore shape, cut-off frequency of filters, or others). To perform Kalman filtration the measured signal is decomposed into the useful signal and noise. The construction of useful signal is based on principles of signals similarity in measurement and reference points. The correlated (similar) components of the signals are reconstructed as a result of filtering procedure discussed above.

Because the Kalman filter is not a simple band pass filter it removes noise without reducing the frequency content of the signal. It is very important from point of view further processing methods (e.a. calculation of the transfer function), because preserves the abilities of the magnetotelluric method to recognize both shallow and relatively deep Earth structures.
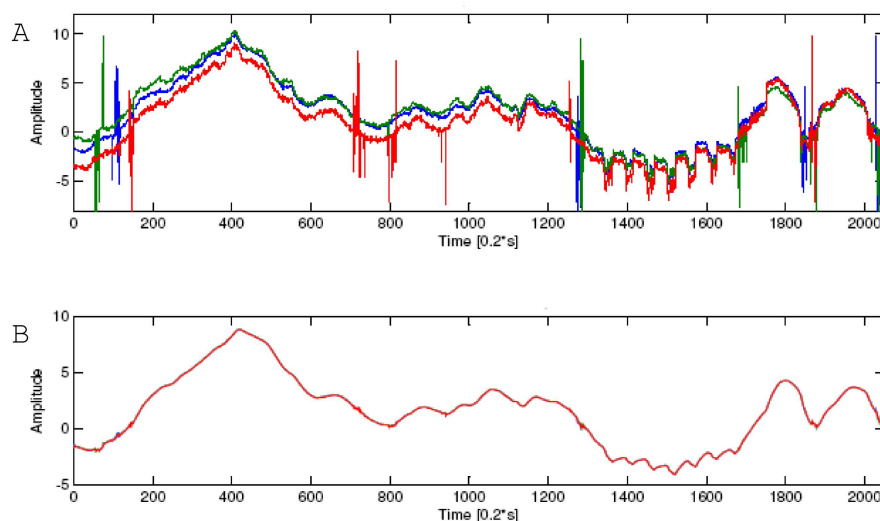
Fig. 4. Results of proposed Kalman filtration in three component variant. Raw (A) and filtred (B) magnetotelluric time series for channels Ex-1 (blue), Ex-2 (green) and Ex-3 (red)
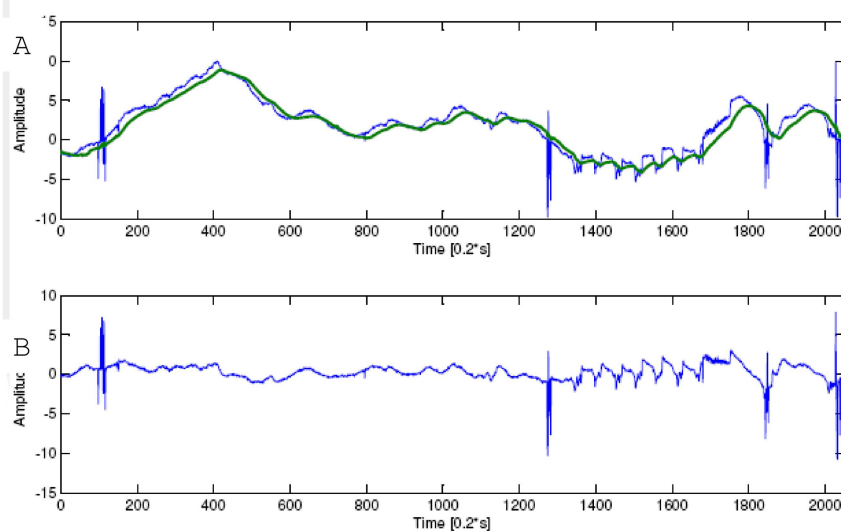
Fig. 5. A) Comparison between original (blue) and filtered (green) Ex-1 time series in case of three component Kalman filtering. B) Amplitude difference between raw and filtered time series

The filter implemented here adapt to local noise level and to the local shape of the signal. The filter parameters (process and measurement noise covariances Q_{kl} and R_{kl}) allows proper adjust the signals in different channels to each other in contrary to standard LMS adaptive filters. What is also important from point of view of practical implementation the Kalman filter proposed in the article is very simple and easy for coding.

6. References

[1]   Bielecka M., Danek T., Wojdyła M., Leśniak A., Baran G.; *Application of artificial intelligence methods to distortions reduction in magnetotelluric measurement data*, 19th International Workshop on Electromagnetic Induction in the Earth, Beijing, China, Vol. 1, 2008, p. 454 (abstract).

[2]   Danek T., Stefaniuk M., Wojdyła M.; *Examples of using singular spectrum analysis in magnetotelluric data processing*, 18th International Workshop on Electromagnetic Induction in the Earth, El Vendrell, Spain, 2006, pp. 2–11.

[3]   Gamble T., Goubau W.M., Clarke J.; *Magnetotellurics with a remote magnetic reference*, Geophysics, 44, 1979, pp. 53–68.

[4]   Kalman R.E.; *A new approach to linear filtering and prediction problems*, Transactions of the ASME–Journal of Basic Engineering, 82 (Series D), 1960, pp. 35–45.

[5]   Leśniak A., Danek T., Wojdyła M.; *Application of Kalman filter to noise reduction of magnetotelluric recordings*, 19th International Workshop on Electromagnetic Induction in the Earth, Beijing, China, Vol. 1, 2008, p. 453 (abstract).

[6]   Simpson F., Bahr K.; *Practical magnetotellurics*, Cambridge University Press, UK, 2005.